

10/18/17 "Champions keep playing until they get it right." Billie Jean King

HW: "Finding Equations of Tangent Lines HW" #1-15 odd

Test 3 on Monday 10/30

AIM: How do we find equations of tangent lines?

Warm Up:

SIT DOWN QUIETLY

# FINDING EQUATIONS OF TANGENT LINES

To find the equation of a tangent line to a function at a specific point:

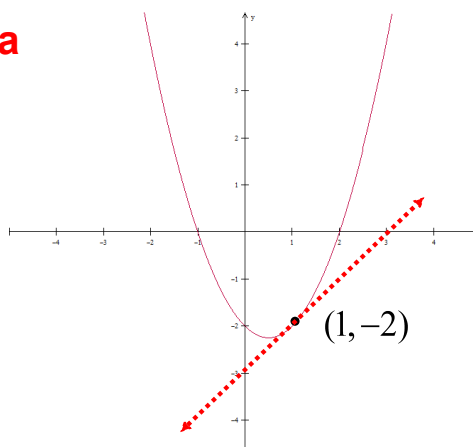
1. Find the y-value of the point on the graph by substituting in the given x-value into the original equation. (function)
2. Find the derivative of  $f$ .
3. Evaluate  $f'(x)$  to get the slope of the graph... THIS MUST BE A NUMBER!
4. Now you have the two basics for any equation of a line, a point and a slope. Use the point-slope formula to write the equation for the tangent line at the given x-value.

$$y_2 - y_1 = m(x_2 - x_1)$$

Point (function) derivative

**EX #1:** The graph of  $y = x^2 - x - 2$  is shown below.

**The slope of a graph at a point is the value of the derivative at that point.**



A.) Find the general derivative of the function.

$$y' = 2x - 1$$

B.) Find the <sup>function</sup> point on the curve when  $x = 1$ .

$$\begin{aligned} y &= x^2 - x - 2 \\ &= 1^2 - 1 - 2 \\ &= -2 \end{aligned}$$

Point:  $(1, -2)$

C.) Find the slope of the tangent line at this point.

$$\begin{aligned} y' &= 2x - 1 \\ &= 2(1) - 1 \\ &= 1 \end{aligned}$$

derivative

Slope of tangent = 1

D.) Write the equation of the tangent line to the graph.

$$y - (-2) = 1(x - 1)$$

OR

$$\begin{aligned} y + 2 &= x - 1 \\ y &= x - 3 \end{aligned}$$

$$0 = x - y - 3$$

E.) Draw the tangent line on the sketch and label the point of tangency.

EX #2: Let  $f(x) = 4x^3 - 5x + 3$ . Find the equation of the line tangent to the graph of  $f$  at  $x = -1$ .

Point:

$$f(-1) = 4(-1)^3 - 5(-1) + 3$$

$$f(-1) = -4 + 5 + 3$$

$$f(-1) = 4$$

$$(-1, 4)$$

Slope:

$$f'(x) = 12x^2 - 5$$

$$f'(-1) = 12(-1)^2 - 5$$

$$f'(-1) = 7$$

$$\text{Slope} = 7$$

$$y - 4 = 7(x - (-1))$$

EX #3: Given the differentiable functions,  $f(x)$  and  $g(x)$ .  
Using the table below,

$x$	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
6	-3	7	2	-4

A.) Find the equation of the line tangent to  $g(x)$   
at  $x = 6$ .

Point:  
 $(6, 7)$

Slope:  
-4

$$y - 7 = -4(x - 6)$$

B.) Find the equation of the line tangent to  $f(x)$   
at  $x = 6$ .

Point  
 $(6, -3)$

Slope:  
2

$$y + 3 = 2(x - 6)$$

## Finding an Equation of a Tangent Line

EX #4: Find the equation of the line tangent to the graph of  $f(x) = x^4 + 2x^2$  at the point where  $f'(x) = 2$ . Round all values to three decimal places.

$$\text{Slope} = 2$$

$$f'(x) = 4x^3 + 4x$$

$$2 = 4x^3 + 4x$$

$$\begin{array}{r} -2 \quad -2 \\ \hline \end{array}$$

$$0 = 4x^3 + 4x - 2$$

$$0 = 2(2x^3 + 2x - 1)$$

$$0 = 2x^3 + 2x - 1$$

$$x = .424$$

$$\text{Point } x = .424$$

$$f(.424) = x^4 + 2x^2$$

$$f(.424) = .392$$

$$y - .392 = 2(x - .424)$$

EX #5: Given  $f(x) = k - x^2$  find the value of  $k$ , constant  
such that the line  $y = -6x + 1$  is tangent to the  
graph of the function.

Slope = -6  
of  
tangent =

$$f'(x) = -2x$$

$$-6 = -2x$$

$$\boxed{x = 3}$$

Meet at the point

$$k - x^2 = -6x + 1$$

$$k - 3^2 = -6(3) + 1$$

$$k - 9 = -18 + 1$$

$$\boxed{k = -8}$$